CBSE Class11 Mathematics

Important Questions

Chapter 10

Straight Lines

1 Marks Questions

1. Find the slope of the lines passing through the point (3,-2) and (-1,4)

Ans. Slope of line through (3,-2) and (-1, 4)

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{4-(-2)}{-1-3}$$

$$=\frac{6}{-4}=\frac{-3}{2}$$

2. Three points $P(h,k)$, $Q(x_1,y_1)$ and $R(x_2,y_2)$ lie on a line. Show that $(h-x_1)(y_2-y_1)=(k-y_1)(x_2-x_1)$

Ans. Since P, Q, R are collinear

Slope of PQ= slope of QR

$$\frac{y_1-k}{x_1-h}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{\cancel{-}(k-y_1)}{\cancel{-}(h-x_1)}=\frac{y_2-y_1}{x_2-x_1}$$

$$(h-x_1)(y_2-y_1)=(k-y_1)(x_2-x_1)$$

3. Write the equation of the line through the points $(1,-1)$ and $(3,5)$

Ans. Req. eq. $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y+1=\frac{5+1}{2}(x-1)$

$-3x+y+4=0$

4. Find the measure of the angle between the lines $x+y+7=0$ and $x-y+1=0$

Ans. $x+y+7=0$

$m_1=\frac{-1}{1}$

$x-y+1=0$

$m_2=\frac{-1}{-1}=1$

Slopes of the two lines are 1 and -1 as product of these two slopes is -1, the lines are at right angles.

5. Find the equation of the line that has y-intercept 4 and is $\perp$ to the line $y=3x-2$

Ans. $y=3x-2$

Slope (m) = $\frac{-3}{-1}=3,$ slope of any line $\perp$ it is $-\frac{1}{3}$

$C=4$

Req. eq. is $y=mx+c$

$y=\frac{-1}{3}x+4$

6. Find the equation of the line, which makes intercepts -3 and 2 on the x and y-axis respectively.

Ans. Req. eq. $\frac{x}{a}+\frac{y}{b}=1$

$a=-3, b=2$

$\therefore \frac{x}{-3}+\frac{y}{2}=1$

2x - 3y + 6 = 0

7. Equation of a line is $3x-4y+10=0$ find its slope.

Ans. $m=\frac{-\text{coff. of } x}{\text{coff. of } y}$

$=\frac{-3}{-4}=\frac{3}{4}$

8. Find the distance between the parallel lines $3x-4y+7=0$ and $3x-4y+5=0$

Ans. $A=3, B=-4, C_1=7$ and $C_2=5$

$d=\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}$

$=\frac{|7-5|}{\sqrt{(3)^2+(-4)^2}}$

$=\frac{2}{5}$

9. Find the equation of a straight line parallel to y-axis and passing through the point

(4,-2)

Ans. Equation of line parallel to y -axis is $x = a.....(i)$

Eq. (i) passing through (-4,2)

$a = -4$

So $x = -4$

$x + 4 = 0$

10. If $3x - by + 2 = 0$ and $9x + 3y + a = 0$ represent the same straight line, find the values of a and b.

Ans. ATQ

$$\frac{3}{9} = \frac{-b}{3} = \frac{2}{a}$$

$$b = -1$$
$$\Rightarrow a = 6$$

11. Find the distance between $P(x_1 y_1)$ and $Q(x_2, y_2)$ when PQ is parallel to the y -axis.

Ans. When PQ is parallel to the y -axis,

Then $x_1 = x_2$

$$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}$$

$$= |y_2 - y_1|$$

12. Find the slope of the line, which makes an angle of 30^0 with the positive direction of

y **-axis measured anticlockwise.**

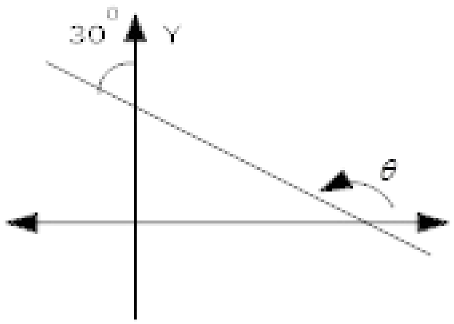
Ans. Let θ be the inclination of the line

$\theta = 120^0$

$m = \tan 120^0$

$= \tan(90+30)$

$= -\sqrt{3}$



13. Determine x **so that the inclination of the line containing the points** $(x, -3)$ **and** $(2, 5)$ **is 135.**

Ans.

$$\frac{5-(-3)}{2-x} = \tan 135$$

$$\left[\begin{array}{l} \because m = \tan\theta \\ m = \dfrac{y_2 - y_1}{x_2 - x_1} \end{array}\right]$$

$$\frac{5+3}{2-x} = -1$$

$x = 10$

14. Find the distance of the point $(4,1)$ from the line $3x-4y-9=0$

Ans. Let d be the req. distance

$$d=\frac{|3.(4)-4(1)-9|}{\sqrt{(3)^2+(-4)^2}}$$

$$=\frac{|-1|}{5}=\frac{1}{5}$$

15. Find the value of x for which the points $(x,-1),(2,1)$ and $(4,5)$ are collinear.

Ans. Let $A(x,-1),B(2,1),C(4,5)$

Slope of AB= Slope of BC

$$\frac{1+1}{2-x}=\frac{5-1}{4-2}$$

$$\frac{2}{2-x}=\frac{4^{2}}{2}$$

$$2-x=1$$

$$-x=-1$$

$$x=1$$

16. Find the angle between the x -axis and the line joining the points $(3,-1)$ and $(4,-2)$

Ans. $m_1=0$ [Slope of x -axis]

$m_2=$slope of line joining points $(3,-1)$ and $(4,-2)$

$= \frac{-2-(-1)}{4-3} = -1$

$\tan\theta = \left|\frac{m_2 - m_1}{1+m_1m_2}\right|$

$\tan\theta = \left|\frac{0+1}{1+0\times(-1)}\right|$

$\tan\theta = 1$

$\theta = 45^0$

17. Using slopes, find the value of x for which the points $(x,-1),(2,1)$ and $(4,5)$ are collinear.

Ans. Since the given points are collinear slope of the line joining points $(x,-1)$ and $(2,1)$ =slope of the line joining points $(2,1)$ and $(4,5)$

$\Rightarrow \frac{2}{2-x} = \frac{2}{1}$

$x = 1$

18. Find the value of K so that the line $2x+ky-9=0$ may be parallel to $3x-4y+7=0$

Ans. ATQ

Slope of 1st line = slope of 2nd line

$\frac{-2}{k} = \frac{-3}{-4}$

$\Rightarrow k = \frac{-8}{3}$

19. Find the value of K, given that the distance of the point $(4,1)$ from the line $3x-4y+K=0$ is 4 units.

Ans. We are given that distance of (4,0) from the line $3x-4y+k=0$ is 4

$$\frac{|3(4)-4(1)+k|}{\sqrt{(3)^2+(-4)^2}}=4$$

$$|k+8|=4\times5$$
$$k=12,-28$$

20. Find the equation of the line through the intersection of $3x-4y+1=0$ $5x+y-1=0$ which cuts off equal intercepts on the axes.

Ans. Slope of a line which makes equal intercept on the axes is -1any line through the intersection of given lines is

$$(3x-4y+1)+K(5x-y-1)=0$$
$$(3+5K)x+y(K-4)+1-K=0$$

$$m=-\frac{(3+5K)}{K-4}=-1$$

$$K=\frac{-7}{4}$$

21. Find the distance of the point $(2,3)$ from the line $12x-5y=2$

Ans. $d=\dfrac{|12x-5y-2|}{\sqrt{(12)^2+(-5)^2}}$

$$d=\frac{|12\times2-5\times3-2|}{\sqrt{169}}$$

$= \frac{|-41|}{13} = \frac{41}{13}$

22. Find the equation of a line whose perpendicular distance from the origin is 5 units and angle between the positive direction of the x -axis and the perpendicular is 30^0.

Ans. $p = 5, \alpha = 30^0$

Req. eq. $x\cos\alpha + y\sin\alpha = p$

$x\cos 30^0 + y\sin 30^0 = 5$

$\sqrt{3}x + y - 10 = 0$

23. Write the equation of the lines for which $\tan\theta = \frac{1}{2}$, where Q is the inclination of the line and x intercept is 4.

Ans. $m = \tan\theta = \frac{1}{2}$ and $d = 4$

$y = \frac{1}{2}(x-4)\ [\because y = m(x-d)]$

$2y - x + 4 = 0$

24. Find the Angle between the x -axis and the line joining the points $(3,-1)$ and $(4,-2)$

Ans. Let $A(3,-1)$ $B(4,-2)$

Slope of $AB = \frac{-2-(-1)}{4-2}$

$= -1$

$\tan\theta = -1$

$\theta = 135 \begin{bmatrix} \text{where } \theta \text{ is the angle which AB makes} \\ \text{with positive direction of } x-axis \end{bmatrix}$

25. Find the equation of the line intersecting the x -axis at a distance of 3 unit to the left of origin with slope -2.

Ans. The line passing through (-3,0) and has slope = -2

Req. eq. is

$y - 0 = -2(x+3)$

$2x + y + 6 = 0$

CBSE Class 12 Mathematics

Important Questions

Chapter 10

Straight Lines

4 Marks Questions

1. If p is the length of the $\perp$ from the origin on the line whose intercepts on the axes are a and b. show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Ans. Equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}-1=0$$

The distance of this line from the origin is P

$$\therefore P=\frac{\left|\frac{0}{a}+\frac{0}{b}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}} \qquad \left[d=\frac{|ax+by+c|}{\sqrt{a^2+b^2}}\right]$$

$$\frac{P}{1}=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\frac{1}{P}=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

Sq. both side

$$\frac{1}{P^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

2.Find the value of p so that the three lines $3x+y-2=0$, $px+2y-3=0$ and $2x-y-3=0$ may intersect at one point.

Ans. $3x+y-2=0........(i)$

$px+2y-3=0.......(ii)$

$2x-y+3=0.......(iii)$

On solving eq. (i) and (iii)

$x=1$, And $y=-1$

Put x, y in eq. (ii)

$P(1)+2(-1)-3=0$

$p-2-3=0$

$p=5$

3.Find the equation to the straight line which passes through the point (3,4) and has intercept on the axes equal in magnitude but opposite in sign.

Ans. Let intercept be a and –a the equation of the line is

$$\frac{x}{a}+\frac{y}{-a}=1$$

$\Rightarrow x-y=a.......(i)$

Since it passes through the point (3, 4)

$3-4=a$

$a=-1$

Put the value of a in eq. (i)

$x-y=-1$

$x-y+1=0$

4.By using area of Δ. Show that the points $(a,b+c),(b,c+a)$ and $(c,a+b)$ are collinear.

Ans. Area of $\Delta=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$=\frac{1}{2}|a(c+a)-b(b+c)+b(a+b)-c(c+a)+c(b+c)-a(a+b)|$

$=\frac{1}{2}.0=0$

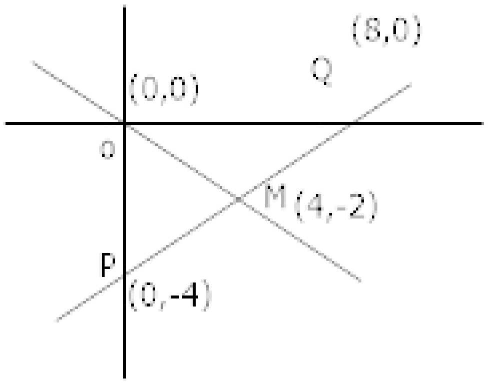
5. Find the slope of a line, which passes through the origin, and the midpoint of the line segment joining the point $p(0,-4)$ and $Q(8,0)$

Ans. Let m be the midpoint of segment PQ then $M=\left(\frac{0+8}{2},\frac{-4+0}{2}\right)$

$=(4,-2)$

Slope of $OM=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-2-0}{4-0}=\frac{-1}{2}$



6.Find equation of the line passing through the point $(2, 2)$ and cutting off intercepts on the axes whose sum is 9

Ans. Req. eq. be $\frac{x}{a}+\frac{y}{b}=1......(i)$

$a+b=9$
$b=9-a$

$\Rightarrow \frac{x}{a}+\frac{y}{9-a}=1$

This line passes through $(2, 2)$

$\therefore \frac{2}{a}+\frac{2}{9-a}=1$

$a^2-9a+18=0$

$a^2-6a-3a+18=0$

$a(a-6)-3(a-6)=0$

$(a-6)(a-3)=0$

$a=6, 3$

$a=6 \quad a=3$

$b=3 \quad b=6$

$\frac{x}{6}+\frac{y}{3}=1 \quad \frac{x}{3}+\frac{y}{6}=1$

7.Reduce the equation $\sqrt{3}x+y-8=0$ into normal form. Find the values p and ω .

Ans. $\sqrt{3}x+y-8=0$

$\sqrt{3}x+y=8......(i)$

$\sqrt{\left(\sqrt{3}\right)^2+(1)^2}=2$

Dividing (i) by 2

$\frac{\sqrt{3}}{2}x+\frac{y}{2}=4$

$x\cos 30^0+y.\sin 30=4.......(ii)$

Comparing (ii) with

$x\cos\omega+y\sin\omega=p$

$p=4$

$\omega=30^0$

8. Without using the Pythagoras theorem show that the points $(4,4),(3,5)$ and $(-1,-1)$ are the vertices of a right angled Δ.

Ans. The given points are $A(4,4),B(3,5)$ and $C(-1,-1)$

Slope of $AB=\frac{5-4}{3-4}=-1$

Slope of $BC=\frac{-1-5}{-1-3}=\frac{-6}{-4}=\frac{3}{2}$

Slope of $AC=\frac{-1-4}{-1-4}=+1$

Slope of $AB\times$slope of $AC=-1$

$\Rightarrow AB\perp AC$

Hence ΔABC is right angled at A.

9.The owner of a milk store finds that, he can sell 980 liters of milk each week at 14 liter and 1220 liter of milk each week at Rs 16 liter. Assuming a linear relationship between selling price and demand how many liters could he sell weekly at Rs 17 liter?

Ans. Assuming sell along x-axis and cost per litre along y-axis, we have two points $(980,14)$ and $(1220,16)$ in x $y$plane

$$y-14=\frac{16-14}{1220-980}(x-980)$$

$$y-14=\frac{\cancel{2}}{{}_{120}\cancel{240}}(x-980)$$

$$120y-14\times120=x-980$$

$$120y-1680=x-980$$

$$x-120y=-700$$

When $y=17$

$x-120\times17=-700\Rightarrow x=1340$ litres.

10.The line through the points (h,3) and (4,1) intersects the line $7x-9y-19=0$ at right angle. Find the value of h.

Ans. Slope of line joining (h,3) and (4,1)

$$=\frac{1-3}{4-h}=\frac{-2}{4-h}$$

Given line is $7x-9y-19=0$

Slope of this line = $\frac{-7}{-9}$

ATQ

$$\left(\frac{-2}{4-h}\right)\times\left(\frac{7}{9}\right)=-1$$

$$\Rightarrow h=\frac{22}{9}$$

11.Find the equations of the lines, which cut off intercepts on the axes whose sum and product are 1 and -6 respectively.

Ans. ATQ $a+b=1.......(i)$

$ab=-6........(ii)$

$b=1-a \quad [from(i)]$

Put b in eq. (ii)

$a(1-a)=-6$

$a-a^2=-6$

$a^2-a-6=0$

$(a-3)(a+2)=0$

$a=3,-2$

When $a=3$

$b=-2$

Eq. of the line is

$$\frac{x}{3}+\frac{y}{-2}=1$$

$$2x-3y-6=0$$

When $a=-2$

$b=3$

Eq. of the line is

$$\frac{x}{-2}+\frac{y}{3}=1$$

$$3x-2y+6=0$$

12. The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines.

Ans. Let the slope of one line is m and other line is $2m$

$$\frac{1}{3}=\left|\frac{2m-m}{1+(2m)(m)}\right|$$

$$\frac{1}{3}=\left|\frac{m}{1+2m^2}\right|$$

$$\pm\frac{1}{3}=\frac{m}{1+2m^2}$$

$$\frac{1}{3}=\frac{m}{1+2m^2}$$

$$2m^2-3m+1=0$$
$$2m^2-2m-m+1=0$$

$$2m(m-1)-1(m-1)=0$$

$$(m-1)(2m-1)=0$$

$$m=1, m=\frac{1}{2}$$

$$\frac{-1}{3}=\frac{m}{1+2m^2}$$

$$-1-2m^2=3m$$
$$2m^2+3m+1=0$$

$$2m^2+2m+m+1=0$$
$$2m(m+1)+1(m+1)=0$$

$$(m+1)(2m+1)=0$$

$$m=-1$$

$$m=\frac{-1}{2}$$

13.Point $R(h,k)$ divides a line segment between the axes in the ratio 1:2. Find equation of the line.

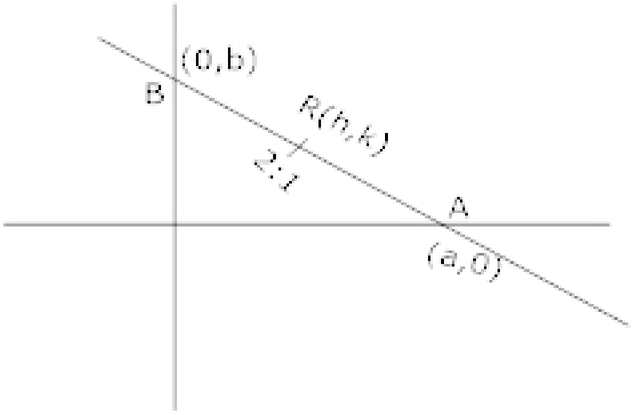
Ans. Let eq. be $\frac{x}{a}+\frac{y}{b}=1$.......$(i)$

It is given that $R(h,k)$ divides AB in the ratio 1:2

$$\therefore (h,k)=\left(\frac{2a}{3},\frac{b}{3}\right)$$

$$\frac{2a}{3}=h$$

$$a=\frac{3h}{2}$$



$$k=\frac{b}{3}$$

$$b=3k$$

Put a and b in eq..........(i)

$$\frac{x}{\frac{3h}{2}}+\frac{y}{3k}=1$$

$$\frac{2x}{h}+\frac{y}{k}=3$$

14.The Fahrenheit temperature F and absolute temperature K satisfy a linear equation. Given that K=273 when F=32 and that K= 373 when F=212 Express K in terms of F and find the value of F when K=0

Ans. Let F along x-axis and K along y-axis

$$K-273=\frac{373-273}{212-32}(F-32)\left[\because y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$$

$$K-273=\frac{100}{180}(F-32)$$

$$K=\frac{5}{9}(F-32)+273$$

15. If three points $(h,0)(a,b)$ and $(0,k)$ lie on a line, show that $\frac{a}{h}+\frac{b}{k}=1$

Ans. Let $A(h,0)B(a,b)$ and $C(0,k)$

Slope of AB = slope of BC

$$\frac{b-0}{a-h}=\frac{k-b}{0-a}$$

$$\frac{b}{a-h}=\frac{h-b}{-a}$$

$$(a-h)(k-b)=-ab$$

$$ak-ab-hk+hb=-ab$$

$$ak+hb=hk$$

$$\frac{ak}{hk}+\frac{hb}{hk}=1$$

$$\frac{a}{h}+\frac{b}{k}=1$$

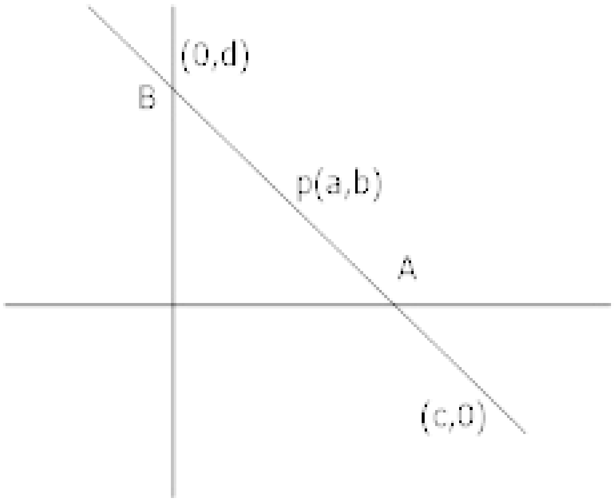
16. $p(a,b)$ is the mid point of a line segment between axes. Show that equation of the

line is $\frac{x}{a}+\frac{y}{b}=2$

Ans. Req. eq. be

$$\frac{x}{c}+\frac{y}{d}=1......(i)$$

P is the mid point



Coordinate of $p=\left(\frac{c}{2},\frac{d}{2}\right)$

$$(a,b)=\left(\frac{c}{2},\frac{d}{2}\right)$$

$$\frac{a}{1}=\frac{c}{2}$$

$$c=2a$$

$$\frac{b}{1}=\frac{d}{2}$$

$$d=2b$$

Put the value of C and D in eq. (i)

$$\frac{x}{2a}+\frac{y}{2b}=1$$

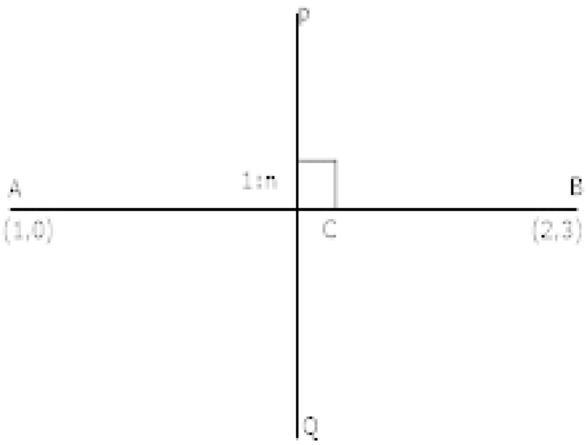
$$\frac{x}{a}+\frac{y}{b}=2$$

17.The line $\perp$ to the line segment joining the points $(1,0)$ and $(2,3)$ divides it in the ratio $1:n$ find the equation of the line.

Ans. Coordinate of $c\left(\frac{2+n}{1+n},\frac{3}{1+n}\right)$

$$m_{AB}=3$$

$$m_{PQ}=-\frac{1}{3}$$



Eq. of PQ is

$$\frac{y}{1}-\frac{3}{1+n}=-\frac{1}{3}\left(\frac{x}{1}-\frac{2+n}{1+n}\right)$$

$$(n+1)x+3(n+1)y-(n+11)=0$$

CBSE Class 12 Mathematics

Important Questions

Chapter 10

Straight Lines

6 Marks Questions

1. Find the values of k for the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$

(a). Parallel to the x-axis

(b). Parallel to y-axis

(c). Passing through the origin

Ans.

(a) The line parallel to x-axis if coeff. Of x =0

$k-3=0$

$k=3$

(b) The line parallel to y-axis if coeff. Of y =0

$4-k^2=0$

$k=\pm 2$

(c) Given line passes through the origin if (0, 0) lies on given eq.

$(k-3).(0)-(4-k^2)(0)+k^2-7k+6=0$

$(k-6)(k-1)=0$

$k=6,1$

2. If p and q are the lengths of ⊥ from the origin to the lines.

$x\cos\theta - y\sin\theta = k\cos 2\theta$, **and** $x\sec\theta + y\operatorname{cosec}\theta = k$ **respectively, prove that** $p^2 + 4q^2 = k^2$

Ans.

$$P = \frac{|0.\cos\theta - 0\sin\theta - k\cos 2\theta|}{\sqrt{(\cos\theta)^2 + (-\sin\theta)^2}} \quad \left[\begin{array}{l}\perp \text{ from origin} \\ \because (0,0)\end{array}\right.$$

$$P = K\cos 2\theta .....(i)$$

$$q = \frac{|0.\sec\theta + 0\operatorname{cosec}\theta - k|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$$

$$= \frac{K}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}}$$

$$= \frac{k\cos\theta.\sin\theta}{\sqrt{\sin^2\theta + \cos^2\theta}} = \frac{1}{2}k.\sin\theta.\cos\theta$$

$$2q = k.\sin 2\theta ......(ii)$$

Squaring (i) and (ii) and adding

$$P^2 + (2q)^2 = K^2\cos^2 2\theta + K^2\sin^2 2\theta$$

$$P^2 + 4q^2 = K^2\left(\cos^2 2\theta + \sin^2 2\theta\right)$$

$$p^2 + 4q^2 = k^2$$

3.Prove that the product of the $\perp$ drawn from the points $\left(\sqrt{a^2 - b^2}, 0\right)$ and $\left(-\sqrt{a^2 - b^2}, 0\right)$ to the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2.

Ans. Let

$$p_1 = \frac{\left|\frac{\sqrt{a^2-b^2}}{a}.\cos\theta - 1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2 + \left(\frac{\sin\theta}{b}\right)^2}} \quad \left[\because \perp \text{ from the points } \sqrt{a^2-b^2}, 0\right]$$

Similarly p_2 be the distance from $\left(-\sqrt{a^2-b^2}, 0\right)$ to given line

$$p_2 = \frac{\left|-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2 + \left(\frac{\sin\theta}{b}\right)^2}}$$

$$p_1 p_2 = \frac{\left|\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\left(-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\right|}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}$$

$$= \frac{\left|\left(\frac{a^2-b^2}{a^2}\right).\cos^2\theta - 1\right|}{\frac{b^2\cos^2\theta + a^2\sin^2\theta}{a^2b^2}}$$

$$= \frac{\left|a^2\cos^2\theta - b^2\cos^2\theta - a^2\right| a^2b^2}{a^2(a^2\sin^2\theta + b^2\cos^2\theta)}$$

$$= \frac{\left|-(a^2\sin^2\theta + b^2\cos^2\theta)\right| b^2}{a^2\sin^2\theta + b^2\cos^2\theta} \qquad \left[\because a^2\cos^2\theta - a^2 = a^2(\cos^2\theta - 1)\right.$$

$$= \frac{(a^2\sin^2\theta + b^2\cos^2\theta)\, b^2}{a^2\sin^2\theta + b^2\cos^2\theta}$$

$= b^2$

4. Find equation of the line mid way between the parallel lines $9x+6y-7=0$ **and** $3x+2y+6=0$

Ans. The equations are

$$9x+6y-7=0$$

$$3\left(3x+2y-\frac{7}{3}\right)=0$$

$$3x+2y-\frac{7}{3}=0.......(i)$$

$$3x+2y+6=0.....(ii)$$

Let the eq. of the line mid way between the parallel lines (i) and (ii) be

$$3x+2y+k=0.......(iii)$$

ATQ

Distance between (i) and (iii) = distance between (ii) and (iii)

$$\left|\frac{K+\frac{7}{3}}{\sqrt{9+4}}\right|=\left|\frac{K-6}{\sqrt{9+4}}\right| \quad \left[\because d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}\right]$$

$$K+\frac{7}{3}=K-6$$

$$K=\frac{11}{6}$$

Req. eq. is

$3x+2y+\frac{11}{6}=0$

5. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1,2) in the line $x-3y+4=0$

Ans. Let $Q(h,k)$ is the image of the point $p(1,2)$ in the line.

$x-3y+4=0......(i)$

Coordinate of midpoint of $PQ=\left(\frac{h+1}{2},\frac{k+2}{2}\right)$

This point will satisfy the eq. ......(i)

$\left(\frac{h+1}{2}\right)-3\left(\frac{k+2}{2}\right)+4=0$

$h-3k=-3.......(i)$

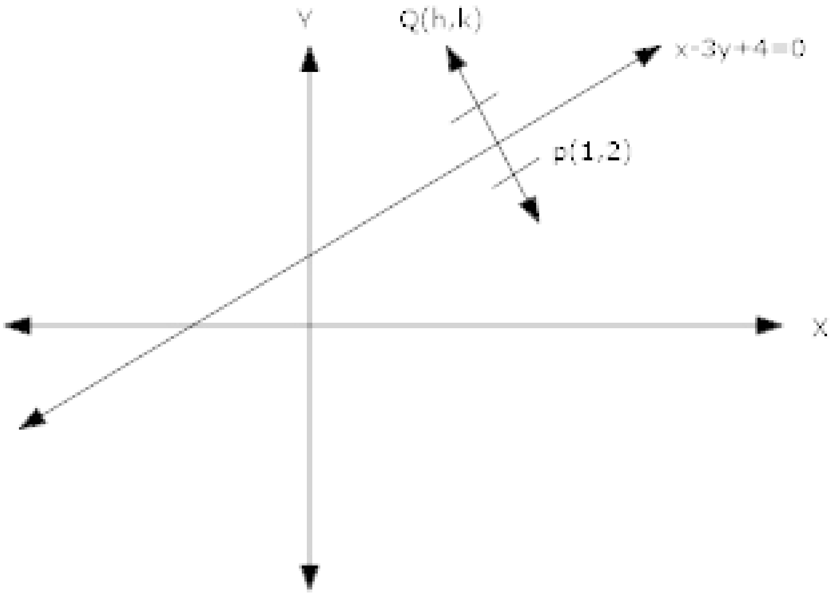
(Slope of line PQ) $\times$ (slope of line $x-3y+4=0)=-1$

$\left(\frac{k-2}{h-1}\right)\left(\frac{-1}{-3}\right)=-1$

$3h+k=5......(ii)$

On solving (i) and (ii)

$h=\frac{6}{5}$ and $k=\frac{7}{5}$



6.A person standing at the junction (crossing) of two straight paths represented by the equations $2x-3y+4=0$ and $3x+4y-5=0$ wants to reach the path whose equation is $6x-7y+8=0$ in the least time. Find equation of the path that he should follow.

Ans. $2x-3y-4=0......(i)$

$3x+4y-5=0......(ii)$

$6x-7y+8=0.......(iii)$

On solving eq. (i) and (ii)

We get $\left(\frac{31}{17},\frac{-2}{17}\right)$

To reach the line (iii) in least time the man must move along the $\perp$ from crossing point $\left(\frac{31}{17},\frac{-2}{17}\right)$ to (iii) line

Slope of (iii) line is $\frac{6}{7}$

Slope of required path $= \frac{-7}{6}$ $[\because m_1 \times m_2 = -1]$

$y - \left(-\frac{2}{17}\right) = \frac{-7}{6}\left(x - \frac{31}{17}\right)$

$119x + 102y = 205$

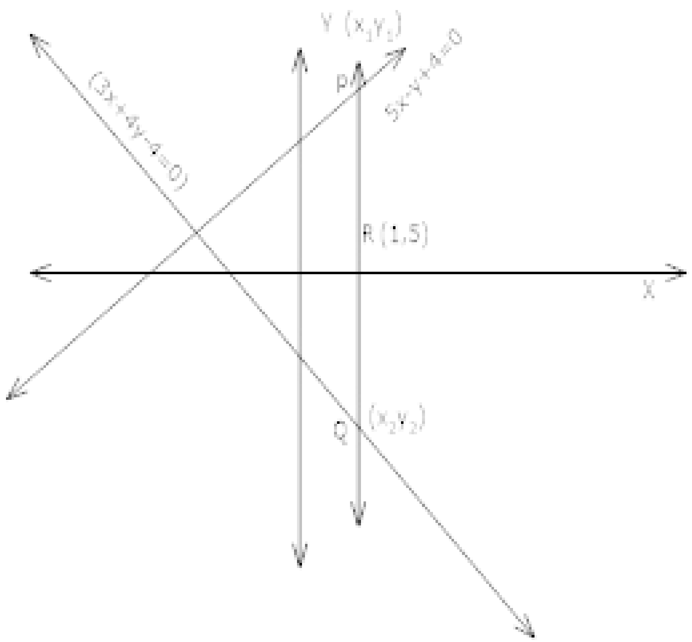
7. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$ obtain its equation.

Ans. $p(x_1 y_1)$ lies on $5x - y + 4 = 0$

$\Rightarrow 5x_1 - y_1 + 4 = 0$

And $Q(x_2 y_2)$ lies on $3x + 4y - 4 = 0$

$3x_2 + 4y_2 - 4 = 0$



On solving

$y_1 = 5x_1 + 4$

$$y_2 = \frac{4-3x_2}{4}$$

Since R is the mid point of PQ

$$\frac{x_1+x_2}{2} = 1, \frac{y_1+y_2}{2} = 5$$

$$x_1 + x_2 = 2, y_1 + y_2 = 10$$

On solving

$$x_1 = \frac{26}{23}, x_2 = \frac{20}{23}$$

And $y_1 = \frac{222}{23}, y_2 = \frac{8}{23}$

Eq. of PQ

$$y - \frac{222}{23} = \frac{\frac{8}{23} - \frac{222}{23}}{\frac{20}{23} - \frac{26}{23}}\left(x - \frac{26}{23}\right)$$

$$107x - 3y - 92 = 0$$

8.Find the equations of the lines which pass through the point $(4, 5)$ and make equal angles with the lines $5x - 12y + 6 = 0$ and $3x - 4y - 7 = 0$

Ans. The slopes of the given lines are $\frac{5}{12}$ and $\frac{3}{4}$

Let m be the slope of a required line

ATQ

$$\left|\frac{m-\frac{5}{12}}{1+m.\frac{5}{12}}\right|=\left|\frac{m-\frac{3}{4}}{1+m.\frac{3}{4}}\right|$$

$$\Rightarrow\left|\frac{12m-5}{12+5m}\right|=\left|\frac{4m-3}{4+3m}\right|$$

$$\frac{12m-5}{12+5m}=\frac{4m-3}{4+3m}$$

$$16m^2=-16$$

$$m^2=-1$$

Neglect

$$\frac{12m-5}{12+5m}=-\frac{4m-3}{4+3m}$$

$$m=\frac{4}{7},\frac{-7}{4}$$

Req. eq. are

$$y-5=\frac{4}{7}(x-4)$$

$$4x-7y+19=0$$

$$y-5=\frac{-7}{4}(x-4)$$

$$7x+4y-48=0$$